National Centre for Excellence in the Teaching of Mathematics

MathsHUBS

Secondary Mathematics Teaching for Mastery: Some themes and key principles

This document is an attempt to distil some important aspects of what teaching for mastery at Key Stages 3 and 4 might look like and results from ideas generated during the planning of the Secondary Teaching for Mastery Specialist programme and discussions with the teachers taking part in the programme.
The intention is to stimulate debate and discussion rather than to offer a definitive definition or set of strategies.

- Mathematics teaching for mastery rejects the idea that a large proportion of people 'just can't do maths'. All students are encouraged by the belief that by working hard at mathematics they can succeed and that making mistakes is to be seen not as a failure but as a valuable opportunity for new learning.
- Facility with procedures and algorithms without a deep and connected understanding does not constitute mastery. Mastery is achieved through developing procedural fluency and conceptual understanding in tandem, since each supports the other.
- Lessons are designed to have a high-level of teacher-student and student-student interaction where all students in the class are thinking about, working on and discussing the same mathematical content. Challenge and the opportunity to deepen understanding of the key mathematical ideas is provided for all.
- Every attempt is made to keep the whole class learning together. Differentiation is achieved, not through offering different content, but through paying attention to the levels of support and challenge needed to allow every student to fully grasp the concepts and ideas being studied. This ensures that all students gain sufficiently deep and secure understanding of the mathematics to form the foundation of future learning before moving to the next part of the curriculum sequence. This prevents students from being left behind.
- For those students who grasp ideas quickly, acceleration into new content is avoided. Instead, these students are challenged by deeper analysis of the lesson content and by applying the content in new and unfamiliar problem-solving situations. If some students fail to grasp an important aspect of the lesson, this is identified quickly and early intervention ensures that they are ready to move forward with the whole class in the next lesson.
- Lesson design identifies the new mathematics that is to be taught, the key points, the difficult points and a carefully sequenced learning journey through the lesson. In a typical lesson, the teacher facilitates whole-class interactive discussion, including active debate and argument based around the tasks offered. Through teacher-student and student-student interaction the teacher encourages demonstration, explanation, exploration, analysis and generalisation (leading to proof where appropriate).
- It is recognised that practice is a vital part of learning, but the practice is *intelligent* practice that aims to, develop students' conceptual understanding and encourage reasoning and mathematical thinking, as well as reinforcing their procedural fluency.
- Teachers use well-crafted examples and exercises which, through careful use of variation (including what to keep the same) focuses students' attention on the key learning point.
- Teachers use carefully chosen examples of key mathematical ideas and concepts (including models, images and diagrams) which emphasise 'what it is' (both standard and non-standard examples) and 'what it is not'.
- Significant time is spent developing a deep understanding of the key ideas and concepts that are needed to underpin future learning. The structures and connections within the mathematics are emphasised, which helps to ensure that students' learning is sustainable over time.
- Key facts such as number facts (including multiplication tables), formulae and relevant theorems, as well as key algebraic techniques, are learnt and practiced regularly in order to avoid cognitive overload in the working memory. This helps students to focus on new ideas and concepts.

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The following framework is helpful in thinking about the various aspects of teaching for mastery. However, they are not separate elements but overlap and inter-relate. For example, choice of important representations is key when considering coherence; using variation to design activities supports both fluency and mathematical thinking.

Using this framework to explore the links between elements will be just as valuable as exploring each element in turn.

1: Coherence

Key Messages:

1. Small steps are easier to take.
2. **Focusing on one key point each lesson** allows for deep and sustainable learning.
3. Certain images, techniques and concepts are **important pre-cursors** to later ideas. Getting the sequencing of these right is an important skill in planning and teaching for mastery.
4. When introducing new ideas, it is important to make connections with earlier ones that have already been understood.
5. When something has been deeply understood and mastered, it can and should be **used in the next steps of learning**.

2: Representation & Structure

Key Messages:

1. The representation needs to clearly show the concept being taught, and in particular the key difficulty point. **It exposes the structure**.
2. In the end, the **students need to be able to do the maths without the representation**
3. A stem sentence describes the representation and helps the students move to working in the abstract ("ten tenths is equivalent to one whole") and could be seen as a representation in itself
4. There will be some key representations which the students will meet time and again
5. **Pattern and structure are related but different**: Students may have seen a pattern without understanding the structure which causes that pattern

3: Variation

Key Messages:

1. The central idea of teaching with variation is to **highlight the essential features of a concept or idea** through varying the non-essential features.
2. **Variation is not the same as variety** – careful attention needs to be paid to what aspects are being varied (and what is not being varied) and for what purpose.
3. When giving examples of a mathematical concept, it is useful to add variation to emphasise:
    a. **What it is (both standard and non-standard examples**);
    b. **What it is not**.
4. When constructing a set of activities or questions it is important to consider what connects the examples; what mathematical structures are being highlighted? Students are encouraged to avoid mechanical practice and, instead, **to practice the thinking process (intelligent practice)**

4: Fluency

Key Messages:

1. **Fluency demands more of students than memorisation** of a single procedure or collection of facts. It encompasses **a mixture of efficiency, accuracy and flexibilit**y.
2. Quick and efficient recall of facts and procedures is important in order for students to keep track of sub-problems, think strategically and solve problems.
3. Fluency also demands the **flexibility to move between different contexts and representations of mathematics**, to recognise relationships and make connections and to make appropriate choices from a whole toolkit of methods, strategies and approaches.

5: Mathematical Thinking

Key Messages:

1. Mathematical thinking is central to deep and sustainable learning of mathematics.
2. Taught ideas that are understood deeply are **not just 'received' passively but worked on by the student**. They need to be thought about, reasoned with and discussed.
3. Mathematical thinking involves:
    a. looking for **pattern** in order to discern **structure**;
    b. looking for **relationships** and **connecting ideas**;
    c. **reasoning logically**, **explaining**, **conjecturing** and **proving**.