Mastering maths mastery

Windhill21

MathsHUBS
Matrix

A Teacher's guide

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Contents

Introduction 3
What is maths mastery? 4
What can you expect to see in maths mastery lessons? 7
Small steps 10
Teaching whole class together 12
CPA (Concrete, Pictorial, Abstract) approach 18
Carefully chosen representations, questions and tasks 21
Opportunities for pupils to think, reason and problem solve 30
Articulating using correct maths vocabulary and speaking in full sentences 32
Differentiation and how to challenge the more able 35
Secure number facts and fluency 44
How we structure lessons at Windhill21 49
How to use a maths mastery approach in a mixed age class 58

Introduction

I have acquired knowledge about teaching for mastery as maths lead in my own school but also through working with other schools as a mastery specialist as part of the Maths Hubs Teaching for Mastery programme.

In these roles, I have found that the essence of the key features of maths mastery is often lost in confusing terminology or the rationale is unclear. I have also been asked on numerous occasions for examples from my own school to help to explain or address different aspects of maths mastery or to explain what maths mastery looks like in practice at Windhill21.

This teacher guide seeks to explain some of the terminology associated with maths mastery (as I have grown to understand it). It may be a simplified view in places but my aim is to master mastery in everyday real life contexts and help to unpick mastery through illustrated examples from our practices at Windhill21 rather than adhere to strict theoretical definitions.

What is maths mastery?

According to most dictionary definitions, to master something is to be able to do it very well.

The term 'maths mastery' has been defined by a number of organisations:

"…mastering maths means acquiring a deep, long-term, secure and adaptable understanding of the subject. At any one point in a pupil's journey through school, achieving mastery is taken to mean acquiring a solid enough understanding of the maths that's been taught to enable him/her move on to more advanced material." **NCETM website 17 October 2018**

A mathematical concept or skill has been mastered when a pupil can represent it in multiple ways, has the mathematical language to communicate related ideas, and can independently apply the concept to new problems in unfamiliar situations.

Mastery is a journey and long-term goal, achieved through exploration, clarification, practice and application over time. At each stage of learning, pupils should be able to demonstrate a deep, conceptual understanding of the topic and be able to build on this over time.

This is not about just being able to memorise key facts and procedures, which tends to lead to superficial understanding that can easily be forgotten. Pupils should be able to select which mathematical approach is most effective in different scenarios. **(TES booklet: A mastery approach to teaching and learning mathematics Teaching for mastery in primary maths updated Aug 2, 2018)**

For me, teaching for mastery means taking an approach where the emphasis is on teaching for understanding so that pupils are able to use what they know to solve what they do not know.

Rationale for maths mastery

Teaching for maths mastery in UK uses elements of maths teaching from high performing jurisdictions such as Singapore and Shanghai. They significantly outperform England in PISA league tables

Rank	*Country*	*Score*
1 (2)	Singapore	564 (573)
2 (3)	Hong Kong (China)	548 (561)
3 (6)	Macao (China)	544 (538)
4 (4)	Taiwan	542 (560)
5 (7)	Japan	532 (536)
6 (1 – as Shanghai)	Beijing-Shanghai-Jiangsu-Guangdong (China)	531 (613 – as Shanghai)
27 (26)	**United Kingdom**	**492 (494)**

Source: https://www.tes.com/news/pisa-glance-global-education-rankings-science-maths-and-reading

One notable difference between these high performing jurisdictions and the UK is the emphasis on teaching for understanding, going deeper and avoiding three tier differentiation. In other words, expecting that all children will be able to master mathematics. This is my driving force – to engender a love of mathematics through engaging activities that lead to deep understanding.

Aims of the national curriculum for primary mathematics

The aims of the primary national curriculum for mathematics is consistent with taking a mastery approach to teaching mathematics. The aims are:

…to ensure that all pupils:

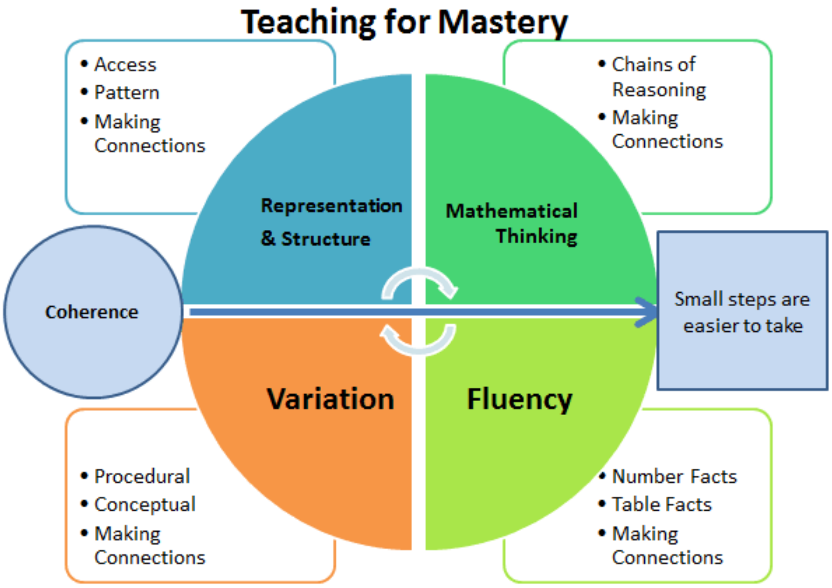
- become **fluent** in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.
- **reason mathematically** by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can **solve problems** by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions. (primary national curriculum 2014)

What can you expect to see in maths mastery lessons?

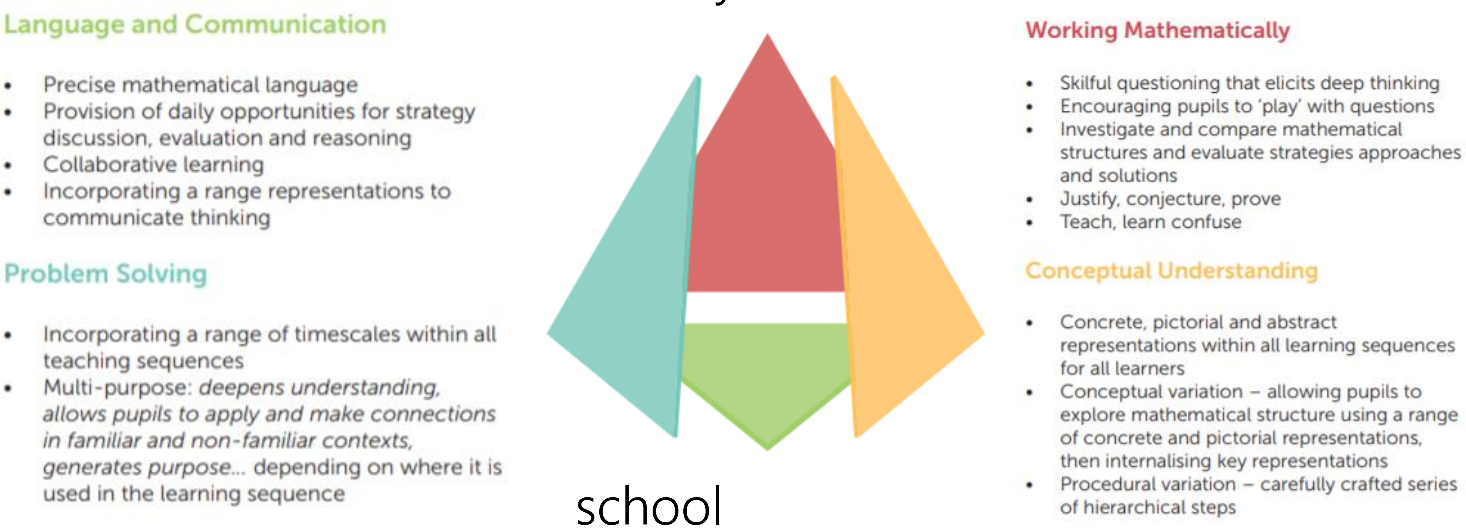
In delivering maths lessons to achieve mastery, you can typically expect to see the following in maths mastery lessons:

- Small steps
- Teaching whole class together
- Ping pong teaching and learning
- Carefully chosen representations, questions and tasks
- CPA (Concrete, Pictorial, Abstract representations)
- Opportunities for pupils to think, reason and problem solve
- Articulating using correct maths vocabulary and speaking in full sentences
- Secure number facts and fluency

Different organisations have summarised these elements as models. The NCETM refers to the '5 Big Ideas':



Herts for Learning (an education company with a not-for-profit ethos) commenced trading in September 2013, as a spin-out from Hertfordshire County Council's



school improvement service. Its model is called a 'tetrahedron of depth'.

https://www.thegrid.org.uk/learning/maths/ks1-2/news/.../pri_maths_news_aut15.pdf

These models are particularly effective in bearing out the interrelatedness of all areas of maths mastery. I will expand on the key concepts of maths mastery in the sections of the booklet that follow. I will also seek to address the elephants in the room (and the questions that I get asked most typically at conferences or teach-meets):

- Teaching the whole together
- Differentiation and how to challenge the more able
- How to structure lessons
- How to teach maths mastery in mixed age classes (especially Year 1-2 classes)

Small steps

Children are taught together and a focussed learning objective is achieved by taking them all through a progression of small, incremental steps. Often these steps start at a point which is a few steps back for all learners to ensure that pre-requisites are in place and understood.

It is obvious that small steps are easier to take when learning a new skill or acquiring new knowledge. By breaking the learning down into small steps, each step becomes successful and children move on from one step to another building on their success and not noticing the tasks getting progressively more difficult or scaffolds being gradually removed. Anxiety is replaced with success. It also naturally closes gaps in knowledge which may never have been exposed and closes gaps which may be different for different children. If the learning is broken down into small steps, children of all abilities are also more likely to make connections to other areas of maths and apply their skills to a range of problems.

Sometimes I am asked why more able children have to go through all the small steps but it has been clear to me in practice that even those who are seemingly more able can have significant gaps in conceptual understanding and just because 'they can' does not mean 'they know why'. Allowing any child to skip any small step is a danger that it too great to risk. Any knowledge gaps for these seemingly more able children will inherently be more difficult to notice in the first place as they could be masked by proficiency in other areas.

Teaching whole class together

Maths mastery teaching is delivered through whole class teaching with mixed ability groupings. The reluctance to adopt this model is a significant barrier to maths mastery take up but it is understandable since most of us had been trained to believe that children should and could be grouped according to 'ability', with teachers providing each group with its own differentiated work. This section should be read in conjunction with the section on differentiation.

When I reflect back on my former practice of grouping by ability in maths I feel guilty because the truth is that I was putting a ceiling on children's potential, creating wider gaps between the most and least 'able' and making 2/3 of my class sit through a teacher input that was not going to be relevant for the questions that I had earmarked for them. Even when we were developing a growth mindset approach to teaching and allowing children to choose their level of challenge (mild, spicy or hot), pupils rarely had the skills to know what level of challenge to choose. In many cases, they were working on completely different learning objectives or difficulty of task was based on larger numbers:

LAs	Mas	HAs
23 + 14 =	213 + 104 =	4652 + 3216 =
61 + 34 =	266 + 732 =	3248 + 6112 =
46 + 23 =	437 + 142 =	2347 + 7432 =
38 + 51 =	334 + 353 =	6402 + 3248 =

Now progression that all pupils would be expected to go through would look something more like this:

213 + 104 =
223 + 105 =
333 + 115 =
334 + 114 =
444 + 114 =
534 + ___ = 668
___ + 434 = 578
_44 + 1_4 = 588
4_4 + _ _ _ = 65_

How many different possibilities can you find?
Can you find them all?
How do you know if you have found them all?

Now I have high expectations of EVERYONE! More on challenge and differentiation which is connected to this in the next section.

However, whole class teaching and mixed ability groupings must be implemented and maintained appropriately. Careful consideration needs to be given to pairings to foster a supportive and collaborative relationship among pupils.

National Curriculum expectations

National curriculum says

The expectation is that the majority of pupils will move through the programmes of study at broadly the same pace. However, decisions about when to progress should always be based on the security of pupils' understanding and their readiness to progress to the next stage. Pupils who grasp concepts rapidly should be challenged through being offered rich and sophisticated problems before any acceleration through new content. Those who are not sufficiently fluent with earlier material should consolidate their understanding, including through additional practice, before moving on.

While this means that children should not typically be accelerated onto content from a year group beyond their own, it has unfortunately incorrectly been misconstrued. I have been told of rapid graspers being held back, being asked to repeat tasks which have already been accomplished while 'waiting' for their peers to catch up. This causes me great concern as opportunities to enrich such children are missed. They should be given rich and sophisticated tasks which allow them to think and reason mathematically in the context of their own year group content (see more on this in the section on challenging the more able and differentiation). Few would argue that children would benefit more from tasks like

4_4 + _ _ _ = 65_ (pitched at Year 3 content level)

rather than

4652 + 3216 = _____ (pitched at Year 4 content level)

Children with special educational needs

Clearly, if you have children with special educational needs that are significantly behind their age related peers then they may not be included in whole class mathe teaching. This is rare as I have seen in my own practice, despite teaching children with specific educational needs.

However, we are not saying that most children will necessarily achieve the same outcomes within a lesson but I have witnessed that in most lessons, almost every child can *access* the same learning, particularly if they are given the relevant scaffolds (see section on differentiation and how to challenge the more able).

Ping pong teaching and learning

Activities and discussions move regularly back and forth between the teacher and the pupils.

→Teacher moves learning on

→ children have discussion or complete a task individually or in pairs

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→ children have discussion or complete task individually or in pairs

Tasks may be carried out by children on whiteboards, in books or simply through oral discussion.

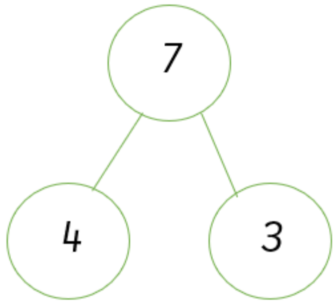
After each task or discussion, the teacher will draw out the key learning points before moving the learning on. The teacher is there to focus the learning and support children to record their ideas, verbalise ideas or deepen understanding.

Learning is very visible during the ping pong teaching and learning. to carry out AfL (identify those children who need immediate intervention in lesson to avoid falling behind).

CPA (Concrete, Pictorial, Abstract) approach

In its simplest form, the CPA approach (used in Singapore maths) enables children to understand a concept though handling physical (concrete) resources before moving onto pictorial representations of items to give meaning to the abstract mathematical symbols.

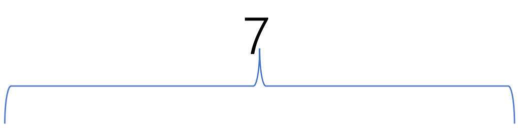
For example, if you are exploring the concept of addition (augmentation) arising from combining items, at the concrete stage you might allow children to handle real crayons and then handle cubes, each one representing a crayon or other concrete object. At the pictorial stage, you use their understanding from the concrete stage to enable them to visualise the problem. This can be done in a variety of ways but typically allows children to work through a problem so that the mathematical notation will become meaningful.

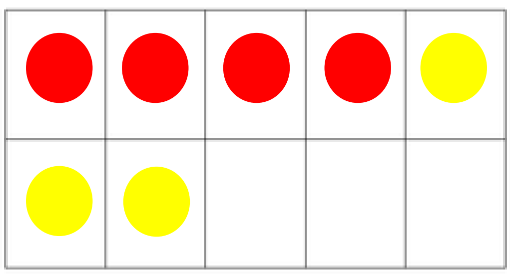


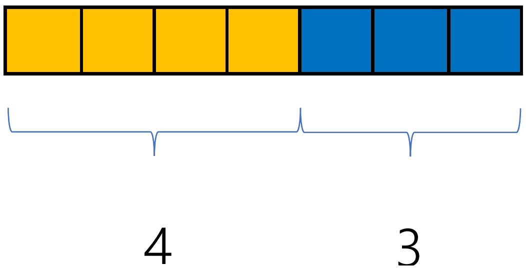
In the abstract stage, pupils make the link with numbers, symbols and mathematical notation e.g. 4 + 3 = 7.

The CPA approach is widely known and used but it is often incorrectly thought of as a linear process with 3 distinct phases. Rather it is a way of developing conceptual understanding where a teacher might develop a concept by

- going back and forth through the CPA stages;
- making links between a number of different concrete and/or pictorial representations or;
- by looking at the concrete, pictorial and abstract representation at the same time.







$7 = \_\_ + 3$

Further, many teachers have rightly concluded, as a result of the CPA approach, that concrete resources are beneficial for all learners (not just those leaners who need additional scaffolds). However, it must also be remembered that once a concept is understood and different representations have been properly explored, it is fine to pursue the abstract without clinging on to concrete or pictorial representations.

Carefully chosen representations, questions and tasks

Teaching for mastery is about deliberate choice by the teacher about representations, questions and tasks to facilitate understanding and to deepen learning. This is underpinned by the teacher being certain about what is to be learnt.

Lots of seemingly complex ideas are associated with carefully choosing questions including variation theory but think it is useful to think of it as:

- choosing specific representations to lead to understanding a new concept by looking at what something is and what it isn't
- being more than just practising a procedure by allowing children to make connections and see patterns
- selecting examples for quality not quantity
- deliberately confusing to test understanding
- deepening learning by expecting children to apply understanding to new areas or to solve problems

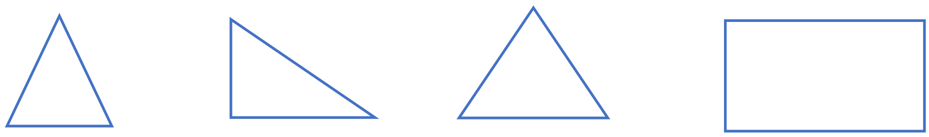
Variation theory

Procedural and conceptual variation

One particular feature of the teaching seen in Shanghai has been the use of teaching with conceptual and procedural variation.

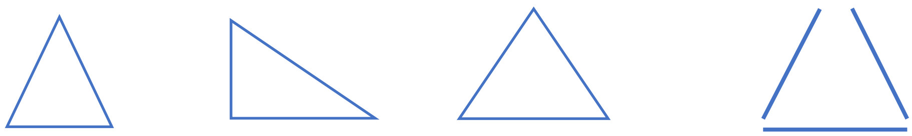
Conceptual variation

Teaching with conceptual variation allows children to understand the concept or idea by comparing different examples of a concept to understand both what it is and what it isn't.



a ☑ b ☑ c ☑ d ☒

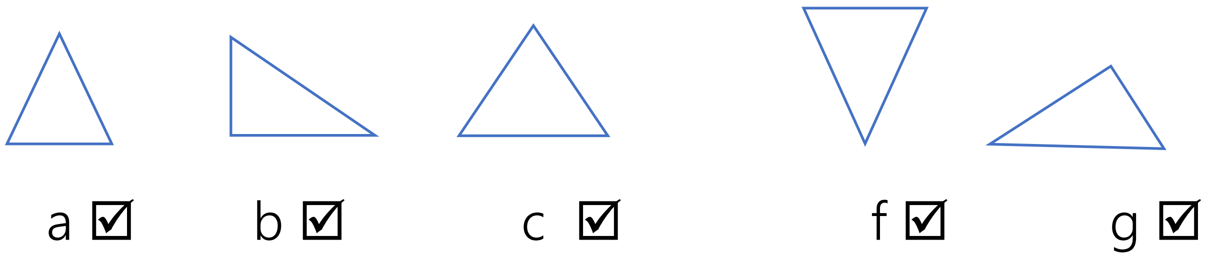
By comparing shapes a to d (d being a non-concept example), a learner can conclude that a triangle has three sides.



a ☑ b ☑ c ☑ e ☒

By comparing shapes a, b, c, e (e being a different type of non-concept example), a learner can conclude that a triangle is a three sided closed shape with three vertices.

However, it is important to introduce <u>non-standard examples</u> to avoid any inadvertent misconceptions



Without including examples f and g, a learner may assume that triangles, especially right angled triangles, need to be in a certain orientation.

In summary, conceptual variation means understanding a concept through the use of standard and non-standard examples and non-concept to understand what something is and what it isn't. This helps pupils to understand the essential and non-essential features of a mathematical concept. For example multiple examples of different triangles will enable pupils to generalise that for a shape to be a triangle it has to be a closed shape with three straight sides and three vertices (i.e. the essential features of a triangle). The non-essential features being side length, angle size, and orientation.

Procedural variation

Procedural variation may also be referred to as 'intelligent practice'. It is about the teacher designing a sequence of related problems. It helps pupils notice what stays the same and what has been changed, allowing connections to be made between one problem and the next. Working on such questions can offer learners an opportunity to explain what is going on, develop reasoning, create their own examples and spot potential errors. It may also allow for generalisations to be made and applied to all situations where the process is used.

Here is a crude example of a set of problems based on the concept of procedural variation:

58 – 24 =___	36 – 25 =___	53 – 22 =___	49 – 24 =___
57 – 25 =___	46 – 24 =___	64 – 23 =___	48 – 25 =___
56 – 26 =___	56 – 23 =___	75 – 24 =___	47 – 26 =___

It could also encourage children to use reasoning to make predictions and create their own number sentences for similar sets of numbers.

The antithesis to intelligent practice is setting out a set of problems based on variety.

58 – 12 =___	86 – 25 =___	73 – 12 =___	48– 24 =___
97 – 25 =___	44 – 24 =___	64 – 33 =___	22 – 21 =___

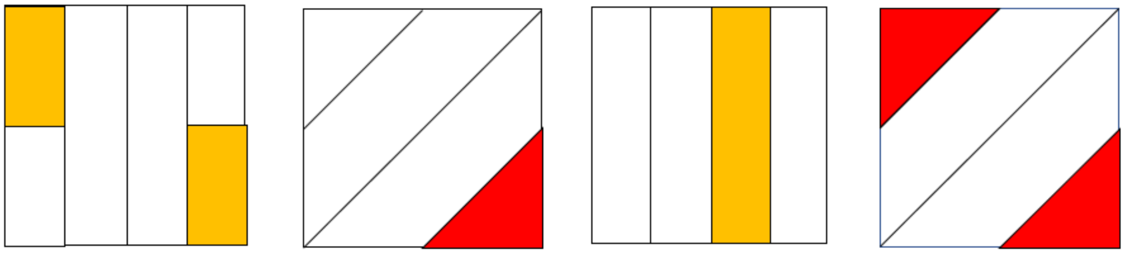
76 – 46 =___	54 – 23 =___	95 – 24 =___	37 – 13 =___

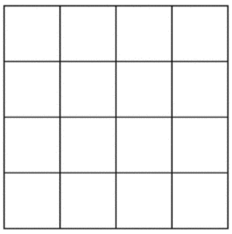
By being random and unconnected, the questions only allow for practice of a procedure without mathematical reasoning, interrogation of structure or any basis to spot potential errors.

Deepening learning

To deepen learning, a teacher can seek to

- structure concepts along the lines of 'Teach, learn, confuse' which is an approach often seen in Shanghai and encompass non-standard or non-examples to 'confuse' pupils, causing them to think and question their own understanding. For example, when exploring fractions, if children really understand the meaning of one quarter, can they explain which of these represent ¼ shaded and reason why.



- Provide opportunities to apply the skill in a new context. This could be as simple as 'turning the question on its head', or providing rich and sophisticated tasks e.g.
  - 36 = ____ - 26
  - Shade ¼ of this shape (how many different ways  can you find?:
  - How many ways could you pack 24 donuts into a rectangular box?

For more on rich and sophisticated tasks (see section on differentiation and challenging the more able).

Further reading and references

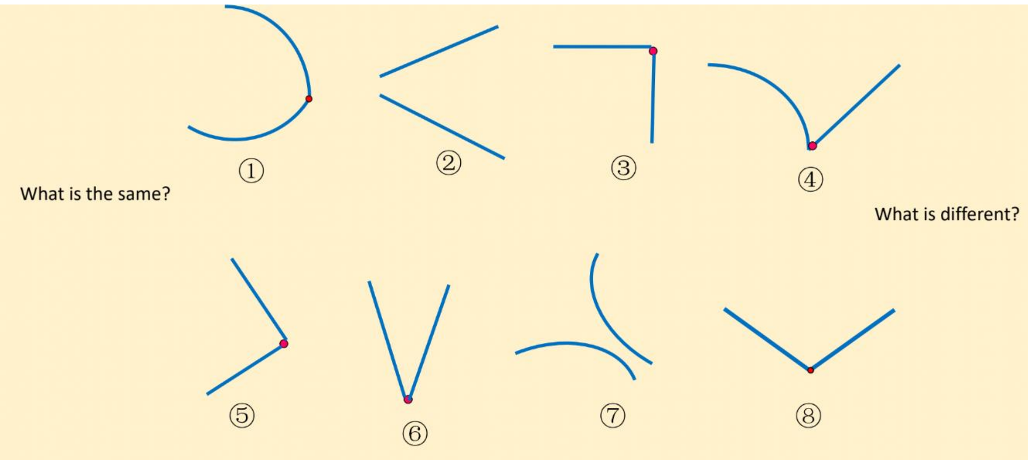
- Teaching with Variation: A Chinese Way of Promoting Effective Mathematics Learning by Lingyuan GU, Rongjin HUNAG and Ference MARTON.

Opportunities for pupils to think, reason and problem solve

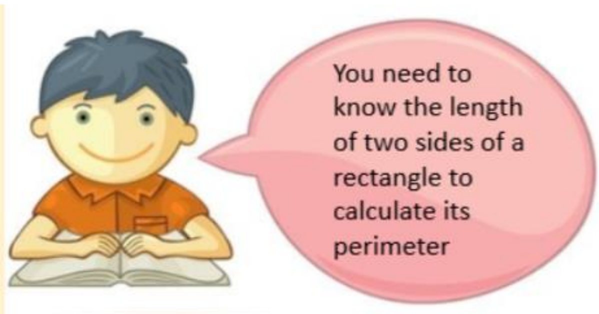
Maths mastery lessons should provide opportunities for pupils to think, reason and problem solve. This is down to carefully crafted questions and specific opportunities for application to new contexts or challenges. Such opportunities can occur at several points in a lesson

Examples from year 4

Focus on thinking – Year 4 revising angles



Focus on reasoning:

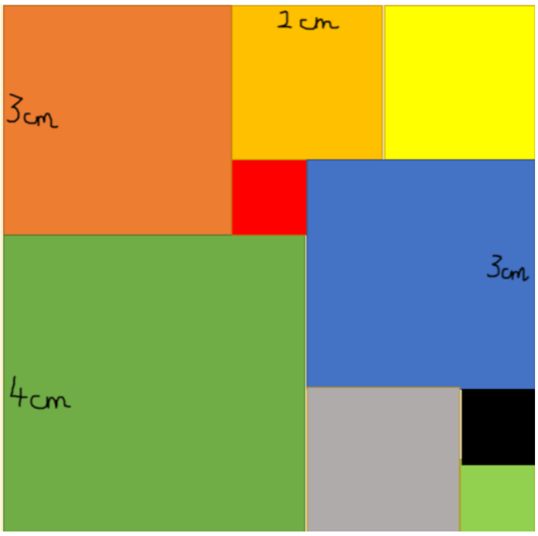


Focus on problem solving:

The perimeter of a rectangle is 40cm. What could the length of each side be? How many possibilities can you find?

Focus on thinking, reasoning and problem solving:

Find the perimeters of all the squares in the image below:



Articulating using correct maths vocabulary and speaking in full sentences

Children's conceptual understanding and mathematical reasoning is significantly improved if they are expected to use correct mathematical terminology (e.g. saying 'digit' rather than 'number') and to explain their mathematical thinking in complete sentences.

Before adopting a mastery approach to teaching maths, my questioning was along the lines of:

> Me: What is 3 times 6?
>
> Pupil: 18
>
> Me: Yes.

It is very tempting to accept a one word answer but these are meaningless on their own. Full sentence answers enables them to articulate and think about what they are saying rather giving mechanical responses. It tests understanding, lays the groundwork for reasoning and assists others in forming constructs based on full sentences. It also enables follow up questions to flow thus extending learning further

Me: What are three 6s?

Pupil: Three 6s are 18.

Me: So if three 6s are 18, how could you this fact to calculate four 6s?

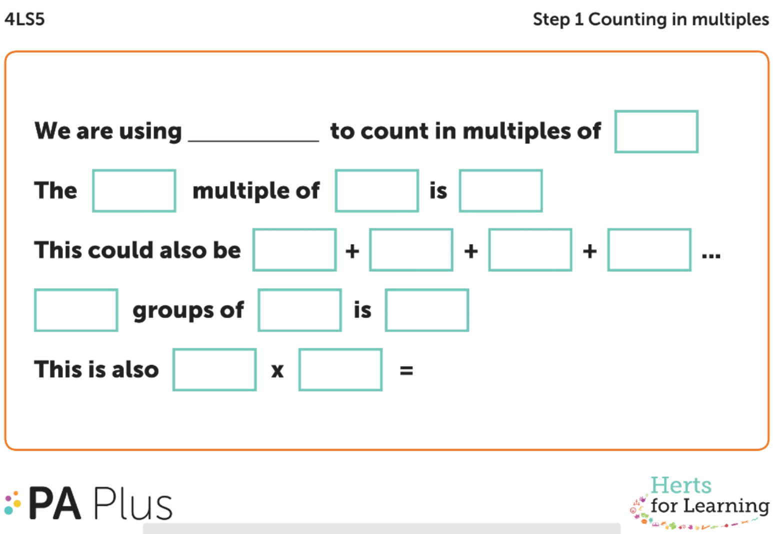
Pupil: Four 6s would be 6 more than three 6s. 6 more than three 6s is 6 more than 18. Four 6s are 24.

Use of sentence stems

Mathematical language is different from everyday language and we need to help our children to develop their fluency with it, to allow them to express their ideas. To assist children in developing the right vocabulary and correct responses, sentence stems can be used. These can be orally modelled to the children by the teacher. Then the teacher then asks individual children to repeat the sentence, before asking the whole class to chant the sentence together. This provides children with a valuable sentence for talking about the maths. Repeated use helps to embed key conceptual knowledge.

Sentence stems can also be given where children fill in the missing parts of a sentence; varying the parts but keeping the sentence stem the same. In constructing stem sentences, the teacher is focussing the children on both the language and the key principles that the children need to be focussed on.

Eg.



Differentiation and how to challenge the more able

Differentiation

When teaching the whole class together, differentiation can be achieved by:

- directing, extending or narrowing teacher questioning
- outcome; children all start on the same task but
    - some complete a greater/fewer number of rich/sophisticated tasks in the same time
    - children complete the same tasks to different degrees of depth
- providing additional scaffolding for those that need it
- providing immediate or same day intervention to those who need it

Evidencing Differentiation

One of the issues that we have faced along our mastery journey is how to evidence differentiation. Essentially, as teachers, we should be able to select books from different children and see that

- all children start at the same point

- some children will complete more/less given tasks in a set time
- children may approach rich/sophisticated tasks in different ways
- children may have a greater / fewer number of responses to rich/sophisticated tasks
- a few children may need additional scaffolds to be able to complete tasks
- a few children may need to be set differentiated questions in the same rich tasks so that they are challenged even further (see section below on Challenge and the More able)

It is the teacher's role to ensure that their books show such evidence of differentiation. At Windhill21, teachers write comments in green pen where needed to show where additional scaffolds or questions are provided. We also review books during and at the end of each lesson to ensure that individual children have met our expectations in terms of the amount of work completed and the extent to which they have approached rich/sophisticated tasks (see further details in section titled, How we Structure Lessons at Windhill21).

Challenge and the More able

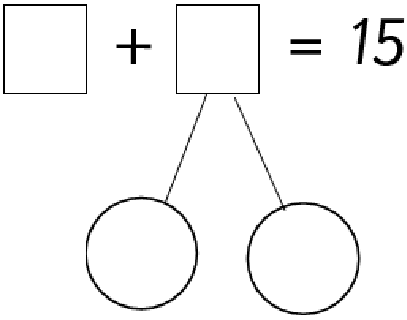
One of the most frequently asked questions that I encounter is "How do you challenge the more able?" Firstly, I dislike the phrase 'more able' as it indicates that there are less able, which again presupposes that some children will succeed where others will fail. The precept of maths mastery is believing that every child will succeed but it is logical to assume that some children will take longer than others to do so. I prefer the National Curriculum description of "rapid graspers" and as we saw previously, rapid graspers should be "challenged through being offered rich and sophisticated problems before any acceleration through new content".

Many teachers claim that they are spending significant time searching the internet for suitable activities and this is hardly surprising since many of the textbooks schemes that I have reviewed offer little challenge or sophisticated problems for rapid graspers

At Windhill21, we are still developing our own practice in this area but generally we write our own challenges based on the following concepts:

- Writing open ended task which encourages thinking, reasoning and problem solving (these should be accessible to all, not just your rapid graspers)
- Differentiating questions for open ended tasks by demanding more from your rapid graspers e.g.
  - Now find ALL possibilities (and explain how you know you have them all)
  - Explain why XXX is not a possible answer,
- Limiting options for open ended tasks to make them more challenging e.g.
  - Can you solve it using all even numbers?
  - What if…

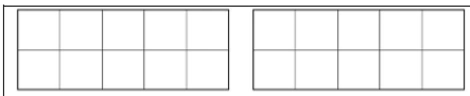
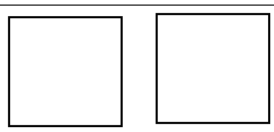
<u>Challenge in Year 1:</u> Context is 'Think 10 strategy' for adding one digit numbers to make teen numbers.



This open ended challenge can be made more challenging as follows:

e.g. what is the smallest / biggest teen number you can make? Can you find all possibilities?

<u>Challenge in Year 1:</u> Context is 'To recognise and make doubles; to represent doubles as an addition calculation.' Here the children show the doubles on the tens frame and as dice patterns before formally writing

		Double ___ is ___ ___ + ___ = ___

This open ended challenge can be made more challenging as follows:

e.g. what is the smallest / biggest double you can make? Is this possible Double ___ is 13? Prove it etc.

Challenge in Year 2: Context using knowledge of place value to add 2 digits numbers efficiently

		+	2		=	7	8	

Find all possibilities

This open ended challenge can be made more challenging as follows:

		+	2		<	7	8

Challenge in Year 2: Multiplication – multiples of 5

___ x 5 < __ x 10

This open ended challenge can be made more challenging as follows:

___ x 5 < 3 x 10

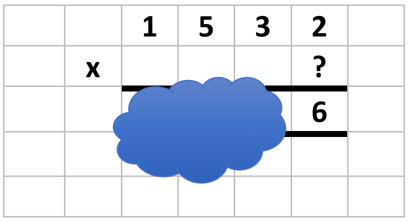
Challenge in Year 3: Adding 3 digit numbers mentally

4_4 + _ _ _ = 65_

This open ended challenge can be made more challenging by differentiating questioning or limiting options:

- How many different possibilities can you find?
- Can you find them all?
- How do you know if you have found them all?
- Can you find a solution that uses all different digits?
- How many possibilities are there if the answer is 657? Can you find a solution to this that uses all even numbers?

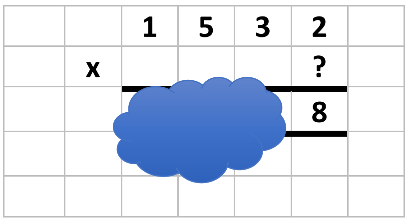
<u>Challenge in Year 4</u>: Context: Short multiplication



Is there more than one possibility?

This semi-open ended challenge can have more or less fixed parameters to make it more challenging e.g.

1. What if the final digit in the solution was 8? How many solutions can you find?



2. Could the final digit in the solution be a 7? Explain your reasoning.

		1	5	3	2	
	x				?	
					7	

3. What if none of the digits in the answer were known?

		1	5	3	2	
	x				?	

- How many solutions can you find?
- Can you find all solutions?
- How do you know that you have them all?

4. What if the answer was 4596? What is the question?

		?	?	?	?	
	x				?	
		4	5	9	6	

- How many solutions can you find?
- Can you find all solutions?
- How do you know that you have them all?

Further reading and references

https://www.hertsforlearning.co.uk/blog/greater-depth-maths-change-rules

https://www.ncetm.org.uk/resources/46830

Secure number facts and fluency

Key facts such as addition facts and multiplication facts (and their inverses) should be learnt to automaticity to avoid overload in the working memory and so as to enable pupils to focus on application, reasoning and problem solving. This is key to achieving mastery.

By way of example, if a child does not know the sum of 7 + 5, then they will struggle to perform standard written methods of addition or use efficient mental methods of addition with larger numbers.   They will also not recognise that 70 + 50 is 120 or that 0.7 + 0.5 is 1.2.

The national curriculum sets out the number facts that children are expected to know and when (see table at the end of this section). However, the way that they are taught is key. Rote learning is not effective as it does not develop understanding of number sense or flexibility or allow for application.

Memorisation of addition facts should be acquired through

- exposure to the structure of addition and different models, contexts, concrete and pictorial representations
- use of strategies to acquire unknown facts from known facts such as near doubles (in other words if I know that double 7 is 14 then I know that 7 + 7 =14 and therefore I know that 7 + 8 will be 15.

Memorisation of multiplication facts should be acquired through

- exposure to the structure of multiplication (e.g. arrays, repeated addition)
- making connections between different tables
- understanding the laws of multiplication (commutative, distributive laws in particular)
- rehearsing and recalling in different contexts (e.g. through playing dice and card games)

Appendix National Curriculum number fact requirements

YEAR 1

• In year 1, pupils should represent and use number bonds and related subtraction facts within 20[1] [emphasis added]
• Notes and guidance (non-statutory) Pupils memorise and reason with number bonds to 10 and 20 in several forms (for example, 9 + 7 = 16; 16 – 7 = 9; 7 = 16 – 9). They should realise the effect of adding or subtracting zero. This establishes addition and subtraction as related operations.
YEAR 2
• Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
• Notes and guidance (non-statutory) Pupils practise addition and subtraction to 20 to become increasingly fluent in deriving facts such as using 3 + 7 = 10; 10 – 7 = 3 and 7 = 10 – 3 to calculate 30 + 70 = 100; 100 – 70 = 30 and 70 = 100 – 30. They check their calculations, including by adding to check subtraction and adding numbers in a different order to check addition (for example, 5 + 2 + 1 = 1 + 5 + 2 = 1 + 2 + 5). This establishes commutativity and associativity of addition.
• Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
• Notes and guidance (non-statutory) Pupils are introduced to the multiplication tables. They practise to become fluent in the 2, 5 and 10 multiplication tables and connect them to each other. They connect the 10 multiplication table to place value, and the 5 multiplication table to the divisions on the clock face. They begin to use other multiplication tables and recall multiplication facts, including using related division facts to perform written and mental calculations.

[1] Note: this means knowing facts for all numbers from 0-20 e.g number facts to 9 and 16 not just to 5, 10 and 20

YEAR 3
• Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables
• Notes and guidance (non-statutory) Pupils continue to practise their mental recall of multiplication tables when they are calculating mathematical statements in order to improve fluency. Through doubling, they connect the 2, 4 and 8 multiplication tables.
Pupils develop efficient mental methods, for example, using commutativity and associativity (for example, 4 × 12 × 5 = 4 × 5 × 12 = 20 × 12 = 240) and multiplication and division facts (for example, using 3 × 2 = 6, 6 ÷ 3 = 2 and 2 = 6 ÷ 3) to derive related facts (for example, 30 × 2 = 60, 60 ÷ 3 = 20 and 20 = 60 ÷ 3).
YEAR 4
• Pupils should be taught to count in multiples of 6, 7, 9, 25 and 1000, recall multiplication and division facts for multiplication tables up to 12 × 12
• By the end of year 4, pupils should have memorised their multiplication tables up to and including the 12 multiplication table and show precision and fluency in their work.
• Notes and guidance (non-statutory) Pupils continue to practise recalling and using multiplication tables and related division facts to aid fluency. Pupils practise mental methods and extend this to three-digit numbers to derive facts, (for example 600 ÷ 3 = 200 can be derived from 2 x 3 = 6). Pupils practise to become fluent in the formal written method of short multiplication and short division with exact answers (see Mathematics Appendix 1). Pupils write statements about the equality of expressions (for example, use the distributive law 39 × 7 = 30 × 7 + 9 × 7 and associative law (2 × 3) × 4 = 2 × (3 × 4)). They combine their knowledge of number facts and rules of arithmetic to solve mental and written calculations for example, 2 x 6 x 5 = 10 x 6 = 60. •

Further reading and references

- https://www.ncetm.org.uk/resources/50006

How we structure lessons at Windhill21

When I first started teaching for mastery, I had taken very literally the idea that the time for all pupils to master a concept could not be predetermined let alone planned to fit within the timings of specific lessons. So at Windhill21, we set about planning learning sequences that pupils worked through and we only moved on to the next sequence once the majority of pupils had completed it. Before long, we realised that this was meeting the needs of few pupils, created gaps and was logistically difficult to manage. As teachers, we operate within a timetable of discrete lessons and we knew that we needed to develop a lesson design structure that was

a. consistent with teaching for mastery; yet
b. rigid enough that it could fit with our timetable constraints; and
c. flexible enough that it could be tweaked to meet the needs of pupils in specific classes / year groups / topics

As a starting point, we adopted the following lesson design model at Windhill21:

		Purpose	Comments
5 mins	Starter task	To recap directly relevant prior learning; or to introduce new topic by means of open discussion around a model / diagram / representation	Practise of unrelated prior learning occurs during afternoon fluency sessions so as not to interrupt the learning episode.
20 minutes	Explore	Ping pong style learning. Usually based on theory of conceptual variation Skilful questioning by teacher and emphasis on oral use of mathematical language	Learning is very visible during the stage. Teacher uses this time to carry out AfL (identify those children who need immediate intervention in lesson to avoid falling behind)
25 minutes	Independent Practice	Carefully chosen tasks. These questions are typically based on procedural variation.	Children work through these tasks which get progressively harder, moving gradually from intelligent practice to tasks which require more problem solving, reasoning and thinking. This section of the lesson allows for children to work at their own pace and differentiation is achieved. (See section below on differentiation and challenging more able)
	Apply	Typically these questions are designed to confuse and test deeper understanding of a concept.	
	Challenge	Typically these questions can be accessed by the majority but open ended nature gives opportunities for demonstrating greater depth thinking.	
50 minutes			

Starter tasks

Starter tasks are quick and entirely relevant to the objective for the lesson. They either recap prior learning because this is going to be built on further in the current lesson or there is an open ended discussion about a model / representation from which the key learning that will become relevant to the lesson is drawn out

We have moved retrieval practice of other prior learning to afternoon maths meetings, where we also recall and rehearse number facts.

Explore

Ping pong style learning to develop conceptual understanding through use of

- Concrete and/or pictorial representations; and/or
- conceptual variation; and
- skilful questioning by teacher and emphasis on oral use of mathematical language. As part of this there is an insistence that children will speak in full sentences using accurate mathematical vocabulary. Often this is bolstered by use of stem sentences.

Independent Practice

Carefully chosen tasks. Questions become progressively harder but small steps enable children to succeed and use what they know to tackle harder questions.

Apply

Typically these questions apply the skill to a new context or area or turn the question on its head to test whether the children have understood a concept rather than simply followed a procedure.

Challenge

Typically these questions are open ended and be accessed by the majority but the nature of the tasks means that there are specifically opportunities for demonstrating greater depth thinking if the tasks can be narrowed or adapted accordingly (as explained in section on differentiation and challenging the more able).

How this lesson structure can be adapted

The weighting of the different elements can be adapted / tweaked for topics, year groups and classes e.g. splitting across two lessons:

Lesson 1

10 mins	Starter task
30 minutes	Explore
10 minutes	Independent practice

Lesson 2

5 mins	Starter task
5 minutes	Explore
10 minutes	Independent practice
	Apply
	Challenge

Marking

It is worth noting that our approach to mathematics teaching at Windhill21 frees teachers to spend time planning lessons rather than hours marking. The mastery approach makes learning more visible so that teachers have a much better idea of where children are during a lesson rather than afterwards. Immediate or same day intervention is provided for children falling behind;

Typically work is marked and corrected during the lesson itself, with more responsibility for self-marking by the children themselves as they get older. This is often done at strategic learning stops during the lesson. Generally,

- patterns / connections between questions are discussed during learning stops

- correct answers are marked in pink and incorrect / incomplete answers are marked in green;
- errors or mistakes are reworked by the children in blue pen;
- teachers review all books after the lesson to
    - ensure that sufficient work has been completed by all children,
    - ensure that differentiation is evidenced (see section on differentiation and challenging the more able) and
    - ensure rapid graspers have been challenged (see section on differentiation and challenging the more able);
- teacher review of books is evidenced by the teacher highlighting the learning objective (in pink or green); and
- as a result of teacher review of books, teachers may provide next step marking (in green pen) to ensure sufficient work is then completed or as evidence of support, differentiation or challenge.

Appendix – Year 3 lesson example

Windhill21 MathsHUBS Matrix

LO: Formal written method of subtraction (no regrouping)

Explore 1

```
  2 7 3
-   2 1
  _ _ 2
```

3 ones minus 1 one is ____ ones.

7 tens minus 2 tens is ____ tens.

2 hundreds minus ____ hundreds is ____ hundreds.

273 – 21 = ________.

The sum of 3 ones and 2 ones is ____ ones.

Explore 2

```
  2 7 3
- 1 2 1
  _____
```

3 ones minus ____ one is ____ ones.

7 tens minus _____ tens is ____ tens.

_____ hundreds minus ____ hundred is ____ hundred.

273 – 121 = ________.

Explore 3

```
  4 _ _ 3
- _ 1 2 1
  2 1 5 _
```

3 ones minus 1 one is ____ ones.

___ tens minus 2 tens is 5 tens.

___ hundreds minus ____ hundreds is ____ hundreds.

4 thousands minus ____ thousands is _____ thousands.

______ - _________ = ________

Explore 4

Jane's calculation	My calculation
354 - 12 = 3 5 4 - 1 2 2 3 2	

Explain what Jane did wrong

____

____

Practise

Solve the following using standard written method

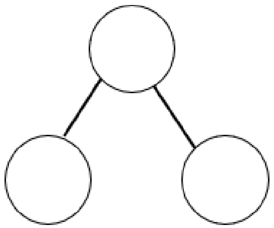
1)

```
  6 7 4
- 3 4 2
  _ _ _
```

2) 674 - 343 =

3) 854 - _____ = 531

Now complete part-part-whole model:



4) Calculate the difference between 987 and 436.

Apply

```
  3 _ 2 3
- 1 6 _ 1
  _ 3 1 _
```

Challenge

Each letter represents a digit.

```
  a c a a
- c b c c
  c c c c
```

How to use a maths mastery approach in a mixed age class

Introduction

In my role as a mastery specialist, I have worked with schools in mixed year group classes. It may be thought of as difficult to see how a mastery approach could be implemented in such a setting, particularly in a mixed 1-2 class. However, I set out below some ideas on how to use a mastery approach to teaching and learning maths in a class which is comprised a mix of Year 1 and Year 2 children.

I informed myself by

- Considering the materials in the further reading and references section
- observing lessons in mixed 1-2 classes, :
- Talking to Cohort 3 mastery specialists who teach in mixed age class schools
- Reviewing planning, mapping documents and exercise books from settings with mixed age classes

Summary

There are two ways to teach maths aligned to the principles of mastery in a mixed 1-2 class:

- Teach each group separately
- Teach both year groups together

Both approaches have limitations and benefits. There are also different ways of teaching each year group separately and different ways of teaching both year groups together. Based on my research, I summarise the main methods below. It is likely that a mixture of approaches is going to be the best model, depending on how well the Year 1 and Year 2 objectives can be aligned in specific maths areas.

Approach 1
TEACH EACH
YEAR GROUP

Option 1a) Teacher to give separate input to each year group on the same day

(Split each maths lesson into two parts:

First half = Part 1

Second half = Part 2

Option 1b) Teacher to give separate input to each year group on different days

(Split each maths lesson into two parts:

Day 1 = Part 1

Day 2 = Part 2

Option 1c) Where there are three mixed year classes, separate children into separate year groups for maths and teach each year group together

	Yr 1	Yr 2
Part 1	Input	Independent task
Part	Independent task	Input

Limitations:

- Need additional adult to support independent task who has not experienced teacher input
- Might need separate space for independent task

Advantages

- Could teach completely separate objectives / topics to each year group

Limitations:

- Only works where there are 3 mixed year group classes
- Class teacher does not teach own class children for maths
- Physical space to house 45 children in one class might be tricky

Advantages

- Could teach completely separate objectives / topics to each year group
- Could use a textbook scheme for input and independent practice
- One teacher plans and delivers maths teaching for year 1 and one

Approach 2
TEACH WHOLE CLASS TOGETHER

National Curriculum objectives for year 1 and year 2 children mapped so it is clear which year 1 objective can be taught in the same lesson as which Year 2 objective

Option 2a) Initial teacher input the same for Year 1s and Year 2s

Same teacher input to Year 1 and year 2 children on the carpet

- acts like revision for Year 2 children so mini challenges / extension tasks may be needed
- could use Year 2 children to EXPLAIN / COACH Y1 children

Year 1 children go to desks for independent practice of Year 1 objective

Year 2 children receive further input from the teacher to

Year 1 children go to desks for independent practice of Year 1 objective

Limitations:

- Need to align objectives for year 1 and year 2
- May need separate space for independent practice in year 1

Advantages

- Useful revision for Year 2 – coaching role can highlight any year 2s with misconceptions

Option 2b) Initial representation the same for Year 1s and Year 2s

Teacher uses the same model /image /representations for input but separate questions are directed to each group reflecting year group objective/expectations. Questions can be directed as

- Split screen questions (left side Year 1 / right side Year 2)
- Oral questions to each group
- A mix of split screen and oral questions

After joint input, all children go to desks for independent practice of questions relevant to

Limitations:

- Need to align objectives for year 1 and year 2
- Need to find representations suitable for both curriculum objectives

Advantages

Suggestions based on research

From the case studies and my research, teaching the whole class together seemed to be the preferred method when there was a single mixed age class. This would seem logical where curriculum objectives can be aligned. The method of teaching the whole class together is dependent on the class itself but it is possible that a mixture of Option 2a) (same input) and Option 2b (same representation) could be used during the course of the year to structure the lessons. It is also possible that even when a majority whole class together approach is used, there may also be times in the year (or certain objectives) that warrant giving separate input to each year group.

Useful resources

- Herts for Learning Essentials maths planning – mixed age Autumn 1 Years 1-2 has some useful ideas for representations that could be used for Option 2b) and for aligning Year 1 and Year 2 curriculum objectives more generally for teaching the whole class together.

- Approach in https://www.hamilton-trust.org.uk/browse/maths/y12/autumn/92234 seems to advocate being flexible and giving separate input when objectives are not aligned and using joint models / representations when aligned

Further reading and references

- https://educationblog.oup.com/primary/mastery-and-whole-class-teaching-in-mixed-year-group-classes
- file:///C:/Users/sarah.pyne.SCHOOLNET.003/Downloads/Teaching_for_Mastery_Mixed_Aged_Classes_Report%20(2).pdf
- https://www.ncetm.org.uk/resources/49022
- https://www.hamilton-trust.org.uk/browse/maths/y12/autumn/92234
- https://www.hamilton-trust.org.uk/browse/maths/y12/spring/92233
- Herts for Learning Essentials maths planning – mixed age Autumn 1 Years 1-2
- http://carmelarchimedesmathshub.org.uk/wp-content/uploads/2017/09/Mixed-Age-Planning-Project-for-Small-Schools-FINAL.pdf