

# Building Flexible, Connected Knowledge: Look For and Make Use of Structure

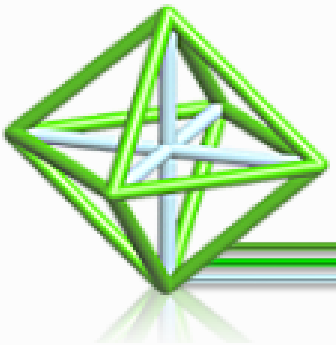
What Does *Math Practice 7* Mean, Anyway?

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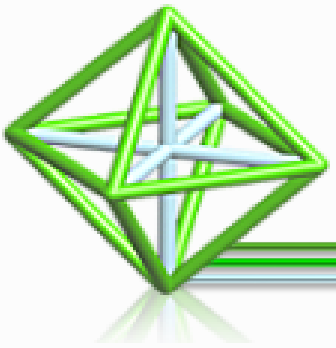
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# Introduce yourself!

- Your name
- Where you work
- Your relationship to math education
- Your current understanding of mathematical structure and/or structural thinking

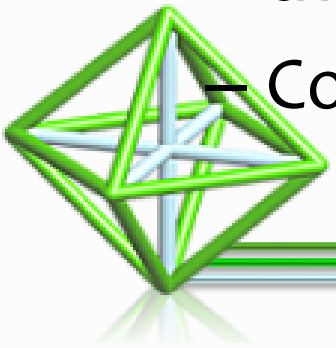


# Purpose

Goal: For all of us to have a deeper understanding of what it means to look for and make of use of structure.

We will reach this goal by:

- Identifying and give examples of structural thinking
- Using a framework to categorize different types of structural thinking
- Considering how this framework might impact our teaching practice



# Why Structure?

Procedural thinking can shortcut deeper understanding of content. Therefore students need to pause and consider the use of mathematical structure. This involves:

- Students working flexibly with content and seeing multiple mathematical possibilities
- Students making connections between different problems revealing broadly applicable mathematical principles



# Do Some Math!

1. Change one digit so that the system has exactly one solution

$$y=2x-15$$

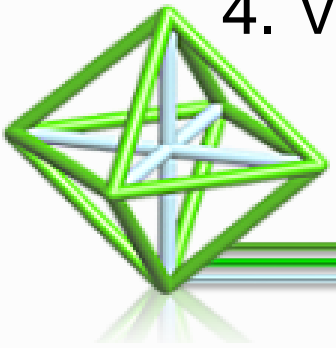
$$y=2(x-5)$$

2. Two lemons and five bananas cost \$1.24. Four lemons and five bananas cost \$1.88. How much does a lemon cost?

3. Solve the following equation:

$$5(x+5)+4(x+5)+2(x+5)=9+10x+50$$

4. Visual patterns problem

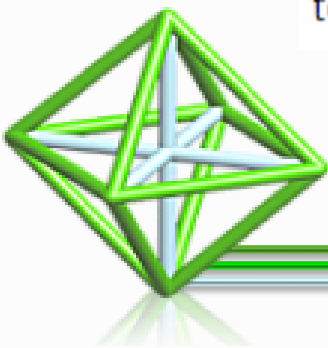


# Math Practices: Commentary and Elaboration (6-8)

## 7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. They might use the structure of the number line to demonstrate that the distance between two rational numbers is the absolute value of their difference, ascertain the relationship between slopes and solution sets of systems of linear equations, and see that the equation  $3x = 2y$  represents a proportional relationship with a unit rate of  $3/2 = 1.5$ . They might recognize how the Pythagorean theorem is used to find distances between points in the coordinate plane and identify right triangles that can be used to find the length of a diagonal in a rectangular prism. They also can step back for an overview and shift perspective, as in finding a representation of consecutive numbers that shows all sums of three consecutive whole numbers are divisible by six. They can see complicated things as single objects, such as seeing two successive reflections across parallel lines as a translation along a line perpendicular to the parallel lines.

Source: Bill McCallum & Illustrative Mathematics  
<https://www.illustrativemathematics.org/practice-standards>

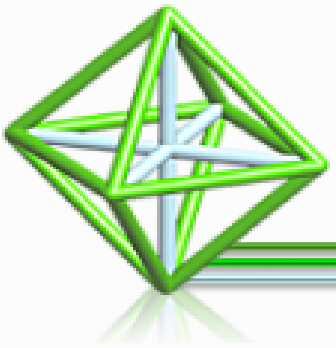


# Structure Within a Problem

Shift perspective to reveal new information that makes the solving process more simple, intuitive, or elegant

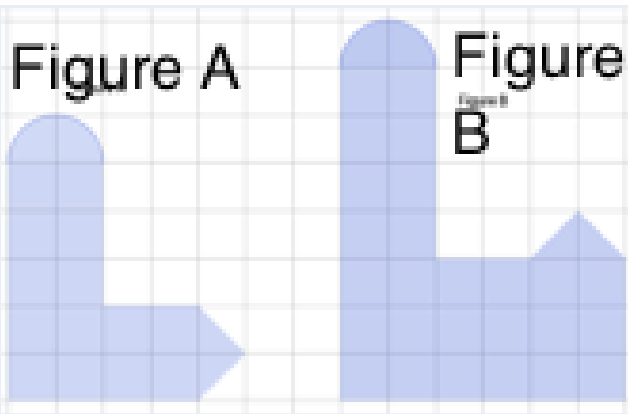
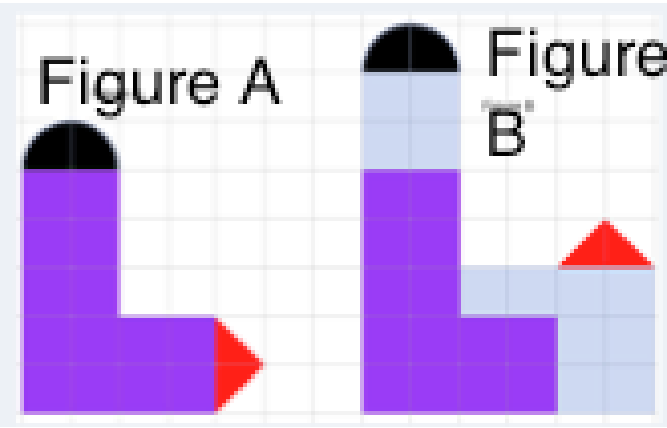
You can shift perspective by...

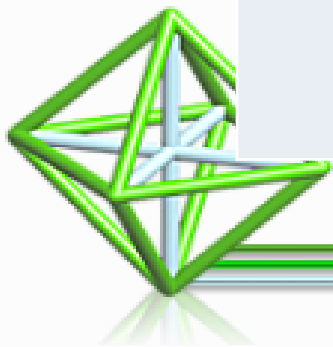
- **Chunking information together**
- **Simplifying or complicating**
- **Using a different representation**



# Chunk Information Together

Look at parts of a situation separately or group information in a way that reveals something not immediately apparent

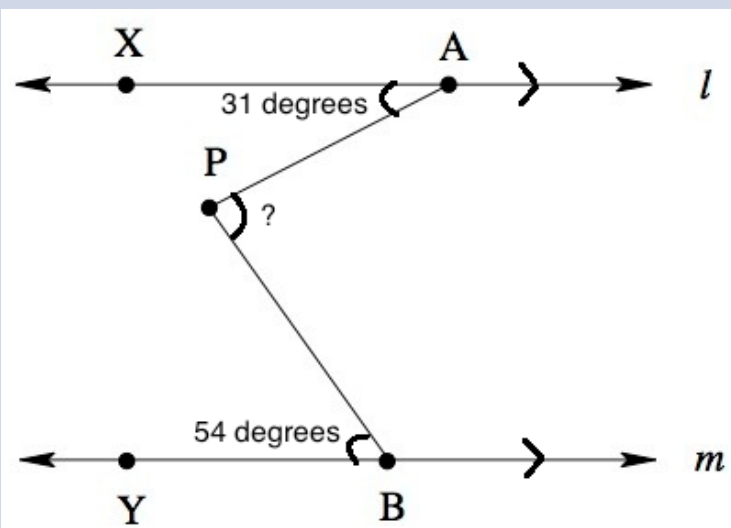
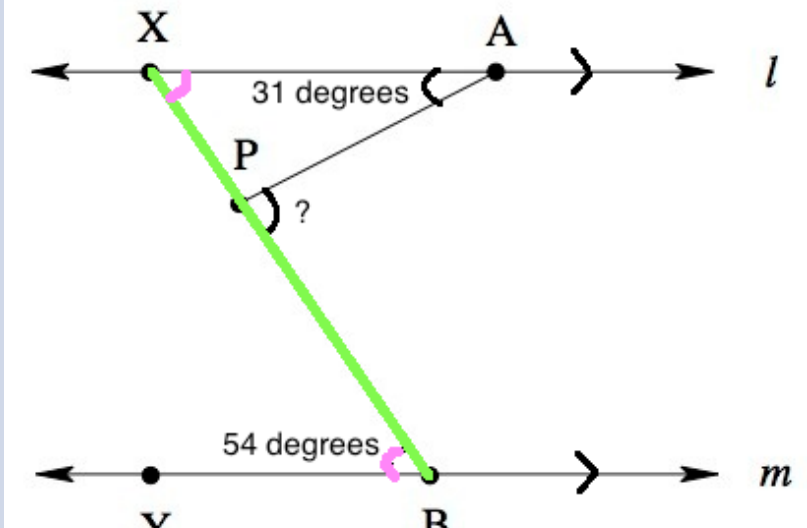
Problem	Structural Thinking
Solve for x: $(x - 3)^2 = x - 3$	$(x - 3)^2 = x - 3$ $\ddot{A}^2 = \ddot{A}$
How much larger is the area of figure B than figure A? 	





# Simplify or Complicate

Add, re-organize, or condense information in a way that reveals something not immediately apparent

Problem	Structural Thinking
<p>? = _____°</p> 	
<p>Adapted From Illustrative Mathematics</p>	
<p>Solve the following quadratic equation:</p> $2x^2 - 19x + 24 = 0$	$2x^2 - 19x + 24 = 0$ $(2x - 3)(x - 8) = 0$



# Use a Different Representation

Look at a mathematical object in a different light to reveal information that was not immediately apparent

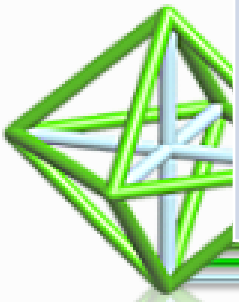
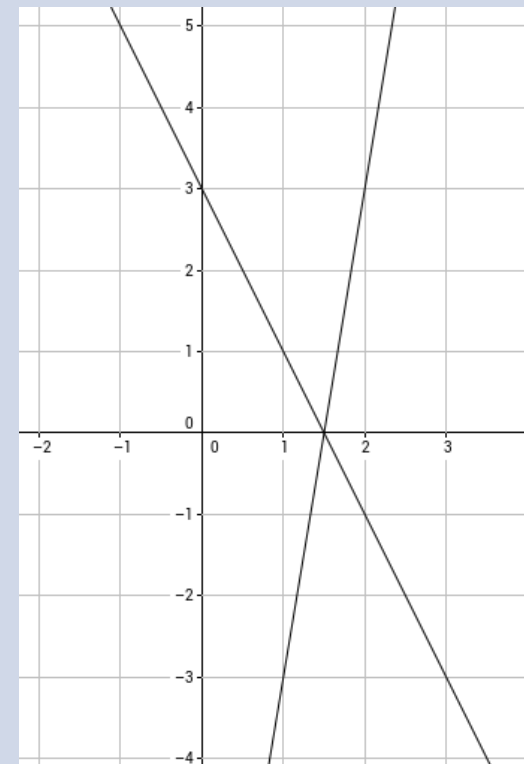
## Problem

What is the solution to the system of linear functions represented by values in this table?

<b>x</b>	<b>-1</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>
<b>y</b>	<b>-15</b>	<b>-9</b>	<b>-3</b>	<b>3</b>	<b>9</b>

<b>x</b>	<b>-3</b>	<b>-2</b>	<b>-1</b>	<b>0</b>	<b>1</b>
<b>y</b>	<b>9</b>	<b>7</b>	<b>5</b>	<b>3</b>	<b>1</b>

## Structural Thinking



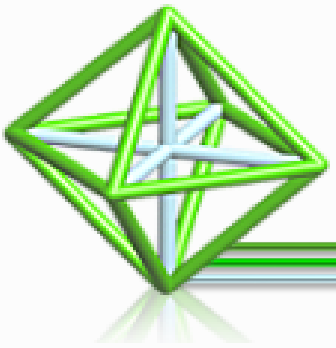
# Structure Within a Problem

Now categorize our examples!

Look back at our list of structural thinking from the first four problems. Try to sort them into our three categories.

- **Chunking information together**
- **Simplifying or complicating**
- **Using a different representation**

Record your examples on your handout.

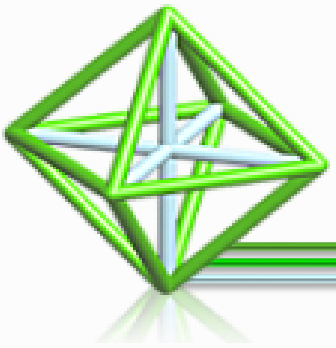


# Structure Between Problems

Use similarities between problems to identify a solution path

You can use similarities between problems by ...

- **Using characteristics of similar problems**
- **Identifying the underlying mathematics**



# Use Characteristics of Similar Problems

Draw upon already-made generalizations and decide if the characteristics of a particular problem fit with a generalization

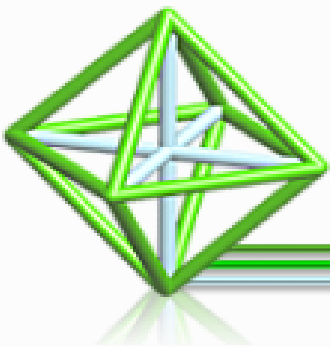
In situations best represented a standard form linear equation, there are often **two rates** and a **constraint**.

Example: Your grandmother made **exactly 240 ounces** of jelly. She has two types of jars: **10-ounce jars** and **12-ounce jars**.

$$10x + 12y = 240$$

x = number of 10-ounce jars

y = number of 12-ounce jars

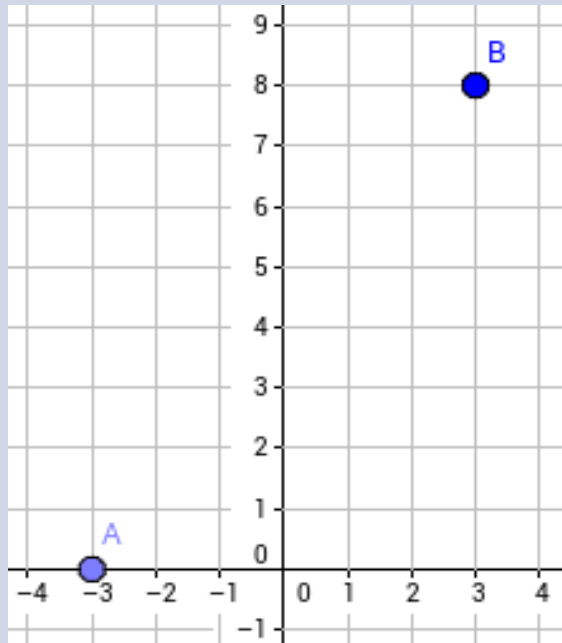


# Identify Underlying Mathematics

Recognize that a problem can be solved using a specific, though not immediately obvious mathematical concept

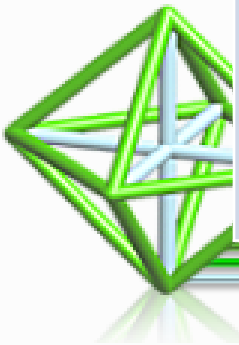
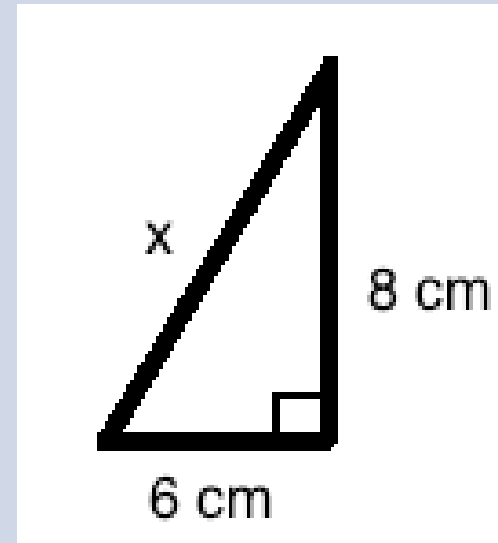
## Problem #1

Determine the distance between points A and B.



## Problem #2

Determine the length of the hypotenuse of the triangle.



# Structure Between Problems

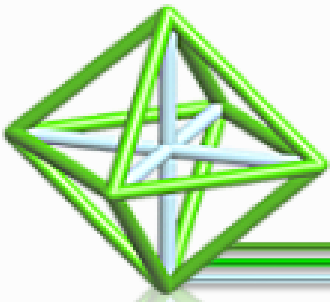
Now go back to our categorizations of the structural thinking for the first 4 problems.

-Are there any you want to move into the between structure categories?

- Can you think of any other examples of between structure thinking that would be useful in solving these problems?

<b>Structure within a problem</b>		
<b>Chunk Information Together</b>	<b>Simplify or Complicate</b>	<b>Use a Different Representation</b>

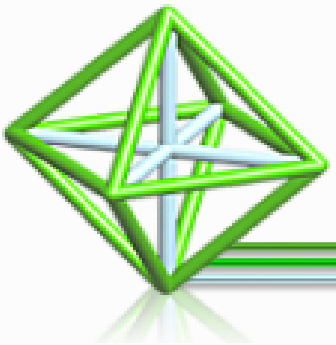
<b>Structure between problems</b>	
<b>Use characteristics of similar problems</b>	<b>Identify underlying mathematics</b>



# Your Turn!

Choose a grade band, and answer the following questions:

- What structure do you see in each task?
- How would you help students see and use this structure?





# A Framework for Structure

## Structure Within a Problem

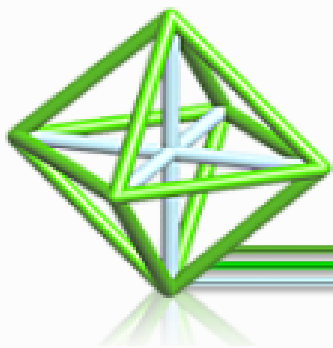
Shift perspective to reveal new information that makes the solving process more simple, elegant, or intuitive.

- Chunk Information Together
- Simplify or Complicate
- Use a Different Representation

## Structure Between Problems

Use similarities between problems to identify a solution path.

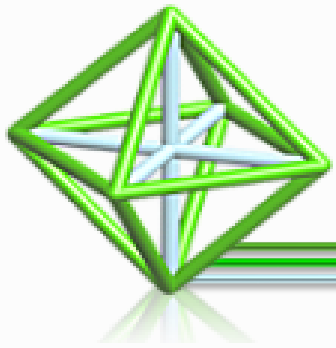
- Identify the Underlying Mathematics
- Use Characteristics of Similar Problems



# Next Steps

Take a minute to reflect and then share with a partner:

1. What is your main take-away today?
2. How might you incorporate these ideas into your teaching?
3. What questions do you still have about structure?



# References

Cuoco, Al. 2015. "Mathematics at the Core: A Tale of Two Standards". Retrieved from <https://www.illustrativemathematics.org/professional-learning>

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